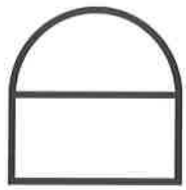
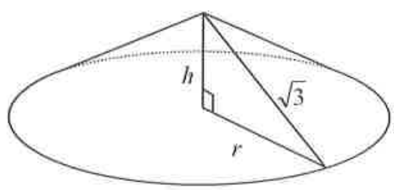
1) A Norman window has the outline of a semicircle on top of a rectangle as shown in the figure. Suppose there is $8+\pi$ feet of wood trim available for all 4 sides of the rectangle and the semicircle. Find the dimensions of the rectangle (and hence the semicircle) that will maximize the area of the window.



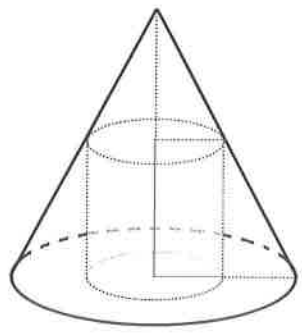
2) You are building a cylindrical barrel in which to put Dr. Brent so you can float him over Niagara Falls. I can fit in a barrel with volume equal 1 cubic meter. The material for the lateral surface costs $18 per square meter. The material for the circular ends costs $9 per square meter. What are the exact radius and height of the barrel so that cost is minimized?

3) A rectangular sheet of paper with perimeter 36 cm is to be rolled into a cylinder. What are the dimensions of the sheet that give the greatest volume?

4) A right triangle whose hypotenuse is $\sqrt{3}$ m long is revolved about one of its legs to generate a right circular cone. Find the radius, height, and volume of the cone of greatest volume. **Note:** $V=\frac{1}{3}\pi r^2 h$.



5) Determine the cylinder with the largest volume that can be inscribed in a cone of height 8 cm and base radius 4 cm.



6) A straight piece of wire 8 feet long is bent into the shape of an L. What is the shortest possible distance between the ends?

7) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 9 - x^2$

8) A closed cylindrical container is to have a volume of $300\,\pi\,\text{in}^3$. The material for the top and bottom of the container will cost \$2 per in^2, and the material for the sides will cost \$6 per in^2. Find the dimensions of the container of least cost.

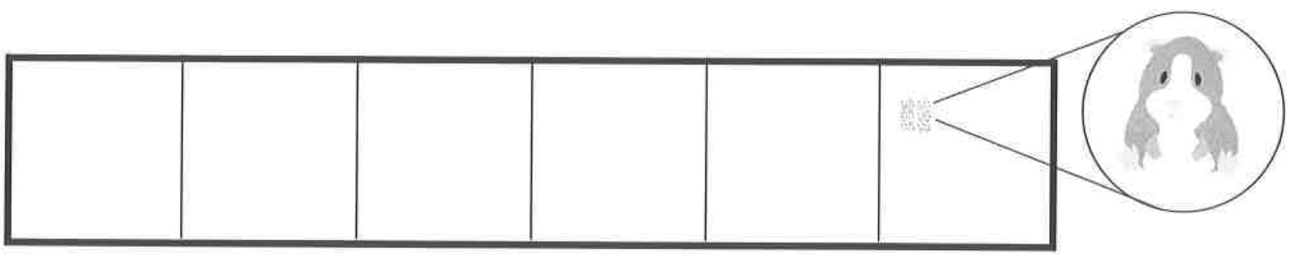
a)Draw a picture, label variables and write down a constrained optimization problem that models this problem. **(5 Pts)**

b) Using calculus, solve the problem in part (a) to find the dimensions.

9) A closed rectangular container with a square base is to have a volume of 300 in^3. The material for the top and bottom of the container will cost \$2 per in^2, and the material for the sides will cost \$6 per in^2. Find the dimensions of the container of least cost.

10) Your dream of becoming a hamster breeder has finally come true.

You are constructing a set of rectangular pens in which to breed your furry friends. The overall area you are working with is 60 square feet, and you want to divide the area up into six pens of equal size as shown below.



The cost of the outside fencing is \$10 a foot. The inside fencing costs \$5 a foot. You wish to minimize the cost of the fencing.

a) Labeling variables, write down a constrained optimization problem that describes this problem.

b) Using any method learned in this course, find the exact dimensions of each pen that will minimize the cost of the breeding ground. What is the total cost?

Optimization Problems

1. Dimensions of rectangle: 2×2.
2. $r = \dfrac{1}{\sqrt[3]{\pi}}$
3. 6×12, $V = \dfrac{216}{\pi}\ cm^3$
4. $r = \sqrt{2}$, $V = \frac{2}{3}\pi$
5. $V = \dfrac{512\pi}{27}$
6. $d = 4\sqrt{2}$
7. $2\sqrt{3} \times 6$
8. $r = \sqrt{450}$, $h = \dfrac{300}{(450)^{2/3}}$
9. $\sqrt[3]{900} \times \sqrt[3]{900} \times \dfrac{300}{(900)^{2/3}}$
10. $464.76